NUMERICAL SOLUTION OF SPECTRAL PROBLEM OF EXTERNAL RADIATION HEATING OF MATTER

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A method is presented for solving spectral problems of radiation in gas dynamics if the problem is independent of at least one Cartesian coordinate. The derivation is given of the corresponding averaged transport equation, and the domain of its applicability is considered. Using this methods a numerical solution is obtained of the heat spectral problem of external radiation heat falling on matter. By comparing it with a similar solution for "gray" matter, the part played by spectral effects [1] its illustrated.

1. Averaged Transport Equation. The radiation transport equation is considered for a planar layer,

$$\mu \frac{dI_\nu}{dr} + \varkappa_\nu' I_\nu = \frac{\varkappa_\nu'}{2} I_{\nu p} \tag{1.1}$$

with the boundary conditions

$$I_\nu(r_0, \mu, t) = I^+(\mu, t, \nu) \quad (\mu \geqslant 0), \qquad I_\nu(r_N, \mu, t) = I^-(\mu, t, \nu) \quad (\mu < 0)$$

In the above μ denotes the cosine of the angle between the direction of the motion of a photon of frequency ν and the r axis; $\varkappa_\nu'$ is the absorption coefficient in which the forced emission has already been taken into account; $I_{\nu p}$ is the equilibrium radiation intensity of a blackbody [2].

The radiation energy flow W can be written as

$$W = \int_0^\infty d\nu \int_{-1}^{1} \mu I_\nu d\nu \tag{1.2}$$

We shall assume that the absorption coefficient is given in the form

$$\varkappa_\nu' = f_1(\nu) f_2(T, \rho) \tag{1.3}$$

where f_1 and f_2 can be arbitrary functions. If in (1.2) a change of variables $\nu = \nu$, $z = \mu / f_1(\nu)$ is carried out and the order of integration inserted, one obtains*

$$W = \int_0^\infty d\nu \int_{-a}^{a} z f_1^2(\nu) I_\nu dz = \int_{-a_+}^{a_+} zI\, dz \tag{1.4}$$

where

$$a = 1 / f_1(\nu), \quad a_+ = \max_\nu 1 / f_1(\nu)$$
$$I = \int_\omega f_1^2(\nu) I_\nu d\nu$$

*The equality (1.4) is valid for any function $f_1(\nu, T, \rho)$.

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TABLE 1

T	$\nu \in [0,5]$	$\nu \in [5,12]$	$\nu \in [12,36]$	$\nu \in [36,72]$	$\nu \in [72,144]$
0.1	10^8	5195	376	14	1.74
0.3	3 703 700	5195	376	14	1.74
1	131 660	3000	376	14	1.74
2	40 316	2737	221	14	1.74
4	18 569	1331	130	10.4	1.74
8	7 242	1137	70	7.2	1.30
12	5 329	825	53	5.9	1.02
16	4 197	344	39	5.3	0.89

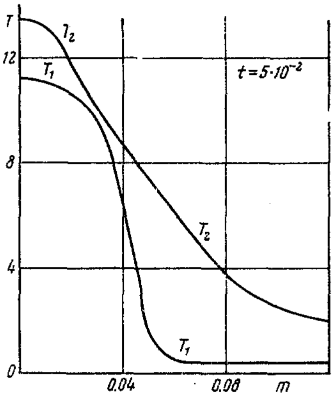

Fig. 1

The integration is carried out over the set $a \geq |z|$.

Multiplying Eq. (1.1) together with the corresponding boundary conditions by $f_1^2(\nu)$ and integrating over the set ω, we obtain the equation

$$zdI/dr + f_2(T,\rho)\, I = f_2 F(T, |z|)/2 \tag{1.5}$$

for the function I and the boundary conditions

$$I^+(t,z) = \int_\omega f_1^2(\nu) I^+(\mu, t, \nu)\, d\nu \quad (z \geqslant 0) \tag{1.6}$$
$$I^-(t,z) = \int_\omega f_1^2(\nu) I^-(\mu, t, \nu)\, d\nu \quad (z < 0)$$

In the above, $F(T, |z|)$ is a monotonically decreasing function of $|z|$ which is given by the formula

$$F(T, |z|) = \int_\omega f_1^2(\nu)\, I_{\nu p}\, d\nu \tag{1.7}$$

The flow W can be determined by solving Eq. (1.5) [1]:

$$W = \int_{-a_+}^{a_+} zI\, dz \tag{1.8}$$

In the case in which $f_1(\nu, T, \rho)$ is a slowly varying function of T, ρ of the order of the length of free motion, one can ignore the term containing the derivative of f_1 with respect to r obtained by multiplying Eq. (1.1) by f_1^2. The corresponding averaged equation for I is written in the form (1.5). The boundary conditions and the right-hand sides of this equation can be determined from (1.6) and (1.7) with $f_1 = f_1(\nu, T, \rho)$. The equality (1.8) becomes now an approximate one.

Let $f_1(\nu, T, \rho)$ be an arbitrary function of ν, T, ρ. By formally integrating the averaged equation (1.5) with respect to z between the limits $-a_+^! \leq z \leq a_+^!$ where $a_+^! = \max 1/f_1(\nu, T, \rho)$, the maximum being taken over ν, T, ρ, one obtains

$$\frac{dW'}{dr} + f_2 U = f_2 B(T, \rho) \tag{1.9}$$

$$W' = \int_{-a_+'}^{a_+'} zI\, dz, \quad U = \int_{-a_+'}^{a_+'} I\, dz, \quad B(T,\rho) = \int_0^\infty f_1(\nu, T, \rho)\, I_{\nu p}\, d\nu \tag{1.10}$$

It follows from (1.9) and (1.10) that in the limiting case of an optically thin heated layer dW'/dr is equal to the divergence of the exact radiation flow dW/dr for any function $f_1(\nu, T, \rho)$.

If the matter is in an equilibrium state with radiation, then $f_1(\nu, T, \rho)$, whatever its form, must be only a slightly changing function within the distances of the order of the length of free motion.

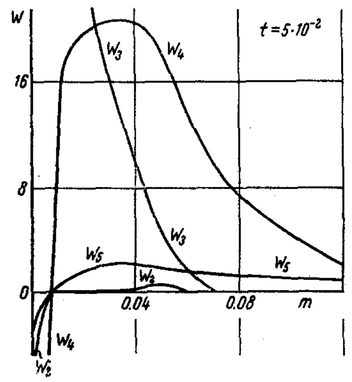

Fig. 2

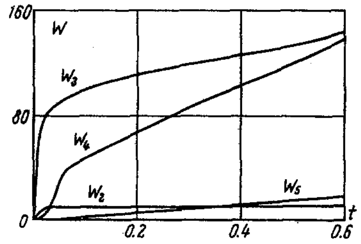

Fig. 3

Therefore, in the "limiting optical states" of matter (an optically thin heated layer and an equilibrium case between matter and radiation), the formally obtained equation (1.5) for an arbitrary f_1 yields results which are identical with the exact results. It is noticed that the widely used diffusion approximation for solving the transport equation in limiting optical states also gives results which are identical with the exact results. In the case of "arbitrary optical states" a qualitatively correct result is always obtained. By analogy with diffusion approximation, it is to be expected that for an arbitrary optical state of matter with arbitrary function $f_1(\nu, T, \rho)$ the averaged equation (1.5) should also give a qualitatively correct result.

Equation (1.5) is of smaller dimensionality than (1.1). It is,moreover, written in the form which is suitable for applying the quasi-diffusion method. The latter enables one to simplify considerably the computation of the flow W [3-5]. It should be mentioned that an averaged equation similar to (1.5) can also be obtained for two-dimensional problems.

Moreover, the proposed averaging can be automatically transferred to the case in which there is no local thermodynamic equilibrium in matter. To obtain the corresponding averaged equation it suffices to set the absorption coefficient and the right-hand side of the transport equation in the form

$$\varkappa = \varkappa_1(\nu)\,\varkappa_2(r,\ t), \qquad I'(\nu,\ r,\ t)$$

where $\varkappa$ and I' are implicit functions of r and t. The derivation of the averaged equation is similar to that of Eq. (1.5).

2. Computation of the Heat Spectral Problem of External Radiation Heating of Matter. The problem is considered of heating an infinite planar layer of matter by external radiation. Problems of this kind arise in laser radiation on matter [6-9]. By assuming that the phase transition has no essential effect on the total process the system of equations describing the motion of the vaporized matter can be written as

$$\frac{dr}{dt} = u, \quad \rho\, dr = dm, \quad \frac{du}{dt} = -\frac{\partial(p+\omega')}{\partial m}, \quad \frac{\partial\varepsilon}{\partial t} = -\frac{\partial W}{\partial m} - p\frac{\partial(ru)}{\partial m} \tag{2.1}$$

$$\varepsilon = \varepsilon(T, \rho), \quad p = p(T, \rho), \quad \varkappa_\nu' = \varkappa'(\nu, T, \rho)$$

$$\mu\frac{dI_\nu}{dr} + \varkappa_\nu' I_\nu = \frac{\varkappa_\nu' I_{\nu p}}{2}, \qquad W = \int_0^\infty d\nu \int_{-1}^{1} \mu I_\nu\, d\mu \tag{2.2}$$

In the above m is the Lagrange coordinate, u is velocity, p is pressure, ω' is artificial viscosity, ε is internal energy.

The gas-dynamics equations (2.1) are written in Lagrange coordinates and the transport equation (2.2) in Euler coordinates. The boundary conditions for the gas-dynamics equations are p = 0 at the left end and u = 0 at the right end. For transport equations the boundary condition can be written as

$$\text{at the left end} \quad I_\nu(r_0(t),\ \mu) = I^+(\mu,\ t,\ \nu)$$
$$\text{at the right end} \quad I_\nu(r_\infty,\ \mu) = 0$$

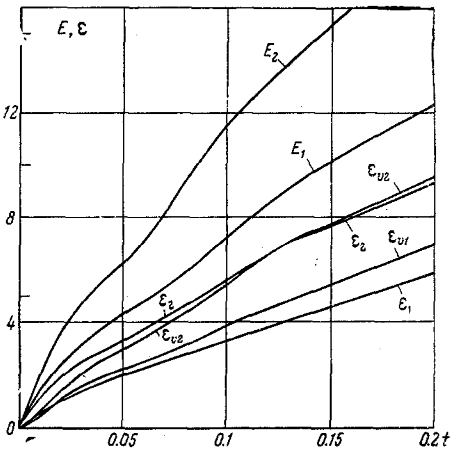

Fig. 4

One takes

$$T(r,\ 0) = u(r,\ 0),\ \ \rho(r,\ 0) = \rho(r)$$

as initial conditions for gas-dynamics equations.

The numerical solution of the system of equations (2.1) and (2.2) is by far more complex than the solving of the problem in the approximation of radiant-heat conduction or in the absence of re-radiation, when the right-hand side of the transport equation is set as equal to zero. To solve numerically the system of equations (2.1) and (2.2) methods described in [4, 5] can be used.

A typical problem will now be considered with the following boundary condition for the transport equation:

$$I^+ = \begin{cases} Ce^t & \nu \in [12,\ 36] \\ 0 & \nu \notin [12,\ 36] \end{cases}$$

and with the state equations close to the state equations of the ideal gas.

The absorption coefficient is assumed to be given by $\varkappa_\nu' = \rho\varphi(\nu,\ T)$ where the values of $\varphi(\nu,\ T)$ are given in Table 1.

In the table the columns were obtained by Planck averaging within the limits of the respective group for the absorption coefficient,

$$\varkappa(\nu,\ T,\ \rho) = \frac{10^4}{T^{0.5}\nu^3}\left(1 - \exp\frac{-h\nu}{kT}\right)$$

Thus the coefficient $\varkappa'(\nu,\ T,\ \rho)$ used in computations is a simulation of the practically important absorption coefficient [2]

$$\varkappa_\nu' = \varkappa_0 \frac{\rho^\beta}{T^\alpha \nu^\theta}\left(1 - \exp\frac{-h\nu}{kT}\right) \tag{2.3}$$

For this problem a five-group approximation is considered as if it were exact. A comparison of an "exact" solution and the solution obtained from the averaged equation shows a good quantitative agreement in spite of the fact that $\varkappa'(\nu,\ T,\ \rho)$ cannot be represented accurately as a product of functions of separable variables.

The results of the computations shown here of the spectral problem were obtained by using the averaged equation (1.5). The solution of this problem was also obtained in a one-group approximation of the gray matter when the absorption coefficient $\varkappa_\nu'$ had been obtained by Planck averaging over the entire spectrum.

It can be seen from the calculations for one-group and also for the spectral modification that at the beginning a substantial part of the system-absorbed radiation energy is used to increase the internal energy of the matter. Subsequently, the share of the kinetic energy increases, there begins an intensive scattering of the matter, and a compression shock wave is formed in front of the heat wave through undisturbed matter. Figure 1 shows characteristic temperature profiles for this state calculated in one-group T_1 and in the spectral T_2 variant.

In the one-group variant the dispersion of matter and the forming of the shock wave begins much earlier than in the spectral case. There is a simple explanation for the latter. The characteristic time τ in which the forming of the shock wave takes place can be determined from the relation

$$\tau = l\ /\ D \tag{2.4}$$

where l is the dimension of the heated region and D is the disturbance transport velocity. So far as the order of quantities is concerned, l is equal to the characteristic length of free motion, which for the spectral variant represents the length of free motion of quanta of the frequency $\nu \in [12, 36]$. This length of free motion exceeds considerably the length of free motion for a one-group variant.

For the regime thus formed (Fig. 1), the matter near the boundary in the spectral variant, unlike the one-group variant, is not in a state of equilibrium with radiation. This is always observed if the radiation is given in a narrow spectral interval and cannot be interpreted as blackbody radiation at some temperature T_0.

In contrast to the one-group variant, the front of the heat wave is more hollow in the spectral variant. One observes the so-called "tongue" of the heat (Fig. 1). To estimate the role of different quanta in the heating of frontal regions, the values of flows are represented in Fig. 2 according to groups. It can be seen from the diagram that the fundamental role as regards the heating function is played by the quanta of the fourth group $\nu \in [36, 72]$; however, in the heating of distant and cool layers an important role is also played by the quanta of the fifth group $\nu \in [72, 144]$. It should be mentioned that hard quanta $\nu \in [36, 144]$ are generated directly in the hot region.

In the course of time the radiation flow falling on the body increases, $W_+ = \pi C e^t$. This leads to an increase in the temperature of the heated zone and consequently to a greater share of the hard-quanta radiation. In Fig. 3, the values of flows are shown according to groups emanating from the body at the instant t. With the rise in power of the radiation there is a greater share of radiation from the fourth and the fifth group. There is hardly any radiation in the first group. It is to be expected that with reduction in the power of radiation, say for example, by $W_+ = \pi C e^{-t}$, the share of radiation of the first and the second group is on the increase. Thus, by varying only the power of the falling radiation without modifying its frequency characteristics a considerable change in the spectral constitution of the outgoing radiation can be obtained.

In Fig. 4, the values are shown of the energy put in the system for the one-group variant E_1 and for the spectral variant E_2. The values are also shown of the internal energy ε and the kinetic energy ε_V for these variants. It can be seen from Fig. 4 that ε and ε_V are almost the same for the spectral variant; their relationship changes, however, for the one-group variant. Thus, the conclusions reached in [6] as regards the existence of an asymptotic ratio of ε and ε_V are confirmed by the example of calculating the spectral problem together with re-radiation.

Let us consider a rapidly increasing external radiation flow,

$$\int_0^{t+\tau} I^+ \, dt \gg \int_0^t I^+ \, dt \tag{2.5}$$

where τ is the characteristic time of dispersion in gas. In this case there is no time for the matter to be set in motion and the share of the kinetic energy should be small. It follows from the inequality (2.5) that the reduced share of the kinetic energy may also be due to an increase in the characteristic dispersion time τ. In turn, τ can be determined from the equality (2.4). When the characteristic length of free motion increases as in the case of proceeding from the one-group variant to the spectral one, the dispersion time τ also increases. An increasing τ leads to a larger share of the internal energy; the latter is confirmed by calculations (Fig. 4).

One should mention here that for some classes of heat problems by radiation on matter the analysis of dimensionality and similarity permits one to select dimensionless combinations of dimensional parameters which determine the solution of the system of equations (2.1) and (2.2). This, in turn, enables one to reduce considerably the number of independent parameters which exert an influence on the solution.

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